Exercise 9

Differentiate both sides of the following equations:

$$x^3 + \frac{1}{6}x^6 = \int_0^x (4 + x - t)u(t)\,dt$$

Solution

Differentiating both sides of the equation with respect to x gives us

$$3x^2 + x^5 = 4u(x) \cdot 1 - (4 + x)u(0) \cdot 0 + \int_0^x \frac{\partial}{\partial x}(4 + x - t)u(t)\,dt,$$

where we used the Leibnitz rule to differentiate the integral. Therefore,

$$3x^2 + x^5 = 4u(x) + \int_0^x u(t)\,dt.$$